Clock Report

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Contents

1 Executive Summary

A pendulum clock was created using an escapement mechanism. The escapement wheel works by pushing the right pallet horizontally which causes the pendulum to oscillate. At the amplitude of oscillation, the left pallet stops the escapement from rotating further. This creates the discrete ticks in the clock as the pendulum oscillates. The presence of the hanging nut creates a gravitational force that helps the escapement wheel overcome the frictional force in the mechanism. The objective of the theoretical analysis was to determine the amount of time required for one full revolution of the escapement wheel for two cases: point-mass and rigid-body. Using Fusion 360's built-in calculation of the center of mass and moment of inertia of the pendulum, the measured times were calculated to be 7.877 and 7.998 seconds for point-mass and rigid-body, respectively. The percent difference between the calculated times were 1.51%. After the pendulum clock was fabricated, the empirical time was measured to be 9.810 seconds. This gives the calculated time from the point-mass analysis a percent error of 19.70% and the rigid-body analysis a percent error of 18%. The most significant sources of error are the existence of friction and air resistance which were both ignored for the point-mass and rigid-body analysis for simplification. Another potential source of error that caused the time to increase is the fabrication of the clock where parts cannot be manufactured to be perfect.

2 Theoretical Analysis

For the pendulum clock analysis, two theoretical methods were used: point-mass and rigid-body. The purpose of using two different methods for our analysis is to examine how using a point-mass versus a rigid-body would affect the period of oscillation and ultimately, the amount of time required for one full revolution of the escapement wheel. The necessary information such as moment of inertia were calculated by Fusion 360 and all the calculations were done using an Excel spreadsheet.

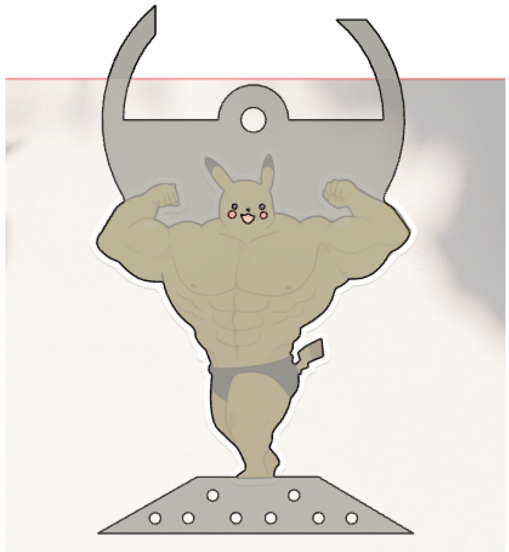

Figure 1: Image of pendulum design in Fusion 360

2.1 Center of Mass

The center of mass of the pendulum was calculated using the area of the pendulum, the thickness of the acrylic, and the density of the acrylic; the area was calculated in Fusion 360.

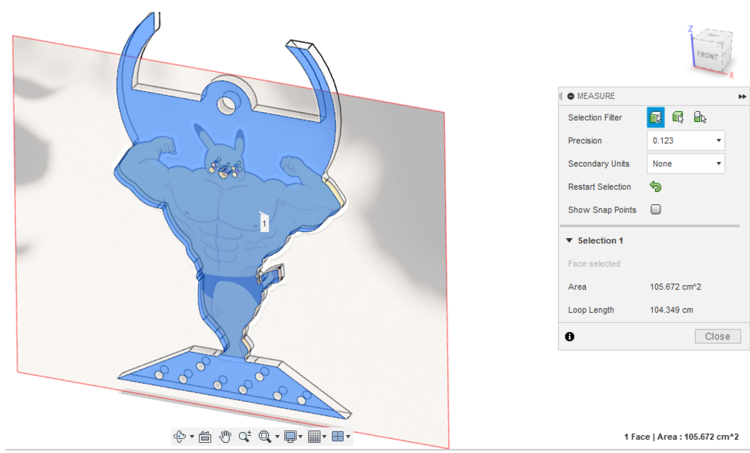

Figure 2: Area of Pendulum in Fusion 360

Using the equation $m = \rho A t$, where m is the mass, ρ is the density of the acrylic, A is the area in Figure 2, and t is the thickness, the mass of the pendulum was calculated to be 79.750 grams in Figure 3.

Acrylic Pendulum Specifications			
Area	A	105.672	cm^2
Thickness	t	0.635	cm
Volume	Vol	67.102	cm^3
Density	p	1.188	g/cm^3
Calculated Mass of Acrylic	M_Calc	79.750	g

Figure 3: Mass Calculation in Excel

The calculated mass was cross-checked with the mass calculated in Fusion 360, which can be seen in Figure 4. Although the calculated mass in Fusion 360 is slightly higher, the values being relatively close is fine as this step is for verification.

Physical	
Mass	79.717 g
Volume	67.102 cm^3
Density	1.188 g / cm^3
Area	277.605 cm^2
World X,Y,Z	0.00 cm, 0.00 cm, 0.00 cm
Center of Mass	-0.001 cm, 0.00 cm, -5.339 cm

Figure 4: Physical properties calculated by Fusion 360

Using the mass of the acrylic, the center of mass of the pendulum with the bolts can be found. Since the mass of one bolt and two nuts was given, the total mass of the pendulum could be calculated by adding the mass of the acrylic and eight sets of bolts. This gives a total mass of 111.75 grams as shown in Figure 5.

Calculate Total Mass of Pendulum				
Mass of one bolt and two nut (Bottom Bolts)	Mb2	4.000	g	given
Number of bolts with 2 nuts	N2	8		
Total mass of pendulum with adapter and bolts and nuts	Mt	111.750	g	calculated

Figure 5: Total Mass Calculation in Excel

Finally, the center of mass of the pendulum with the bolts can be calculated. Using the equation $\frac{\sum_{i=1}^{N} m_1 x_1}{M}$ [1], where m is the mass of individual objects, x is its distance from the pivot point, and M is the total mass. The center of mass was calculated to be 7.867 centimeters (shown in Figure 6).

Calculate Center of Mass of Pendulum with Bolts				
Length to Center of Mass of Bolt 1	L_bolt1	14.379	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 2	L_bolt2	14.379	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 3	L_bolt3	14.369	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 4	L_bolt4	14.369	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 5	L_bolt5	14.369	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 6	L_bolt6	14.369	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 7	L_bolt7	13.553	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 8	L_bolt8	13.553	cm	distance from pivot point to bolt
Length to Center of Mass of Pendulum with nuts and bolts	Lcom	7.867	cm	
Length to Center of Mass of Pendulum with nuts and bolts **In Meters**	Lcom_meter	0.079	**meters**	

Figure 6: Center of Mass Calculation in Excel

With the center of mass calculated, the analysis will now diverge into two theoretical methods: point-mass and rigid-body.

2.2 Point-Mass Analysis

For the point-mass analysis, the assumption was that the pendulum clock was concentrated at a single point, which is a point-mass. Another assumption was that the angular displacement of the pendulum would be relatively small that is under 30 degrees. Similarly, other assumptions were made like ignoring friction and air resistance, the connection between the point-mass and pivot point is not able to stretch and is mass-less, along with the assumption that the frequency of the pendulum doesn't affect the escapement wheel. The reasons for these assumptions were to make the calculations much simpler. For example, by assuming the angular displacement as being under 30 degrees, we could make the assumption that $sin(\theta) = \theta$. By making these assumptions, the calculations were simple and the angular frequency was calculated to be $\omega = \sqrt{g/l}$ [1], where g is the acceleration due to gravity and l is the distance from the point-mass to the pivot point.

As an intermediate verification to check if our pendulum acted like a point mass, the center of mass was calculated experimentally by balancing the pendulum on a finger as shown in Figure 7. The center of mass of just the pendulum without the nuts and bolts was approximately 5 centimeters. This step is used as a verification to determine if the center of mass calculated in Fusion 360 in Figure 4 makes sense, to which it does.

Figure 7: Finger Balance to Find Center of Mass

Using the equation $\omega = \sqrt{g/l}$, the angular frequency was calculated to be 11.167 radians per second. With this value, the period of oscillation was calculated to be 0.563 seconds using the equation $T = \frac{2\pi}{\omega}$. Since the escapement wheel has 14 teeth and there is one period occurs for each teeth, the calculated time of one revolution of the escapement wheel was found to be 7.877 seconds as shown in Figure 8.

Calculate Natural Frequency and Timing using Point Mass Assumption			
Gravitational Constant	g	9.810	m/s^2
Natural Frequency in radians/sec	nat_freq_rad_sec	11.167	rad/s
Natural Frequency in Hz	nat_freq_hz	1.777	Hz
Period of Oscillation	period	0.563	s
Number of teeth of escapement wheel	nteeth	14	
Calculated time of one revolution of escapement wheel	time_calc	7.877	s

Figure 8: Time Calculation for Point-Mass in Excel

2.3 Rigid-Body Analysis

For the rigid-body analysis, the assumptions used were neglecting friction and air resistance, small angle of oscillation, and the escapement wheel not affecting the frequency of the pendulum. With these assumptions, the angular frequency was calculated to be $\omega = \sqrt{\frac{mgl}{I}}$ [1], where m is the mass of the pendulum, g is the acceleration due to gravity, l is the distance from the center of mass to the pivot point, and I is the rotational inertia about the pivot point.

Moment of Inertia at Origin (g cm^2)	
Ixz	10.822
Iyx	0.00
Iyy	4667.642
Iyz	0.00
Izx	10.822
Izy	0.00
Izz	597.313

Figure 9: Moment of Pendulum in Fusion 360

The moment of inertia about the pivot was needed to be calculated. The moment for the acrylic body was calculated in Fusion 360 as seen in Figure 9 and each of moments for the bolt-nut sets were calculated by hand using the equation $\sum_{i=1}^{N} m_1 r_1^2$ [1], where m is the mass of the individual bodies and r is the distance from the mass to the pivot point. This value was calculated to be 7130.359 gram-centimeters squared; using this value and the equation $\omega = \sqrt{\frac{mgl}{I}}$, the angular frequency was calculate to be 10.998 radians per second. The calculated time of one revolution of the escapement wheel was found to be 7.998 seconds as shown in Figure 10.

Calculate Natural Frequency and Timing using Rigid Body Assumption				
moment of inertia of acrylic pendulum	I_a	597.313	g*cm^2	
moment of inertia of bolt 1	I_bolt1	877.107	g*cm^2	
moment of inertia of bolt 2	I_bolt2	877.107	g*cm^2	
moment of inertia of bolt 3	I_bolt3	847.974	g*cm^2	
moment of inertia of bolt 4	I_bolt4	847.974	g*cm^2	
moment of inertia of bolt 5	I_bolt5	713.958	g*cm^2	
moment of inertia of bolt 6	I_bolt6	713.958	g*cm^2	
moment of inertia of bolt 7	I_bolt7	827.483	g*cm^2	
moment of inertia of bolt 8	I_bolt8	827.483	g*cm^2	
total moment of inertia	I_total	7130.359	g*cm^2	
total moment of inertia in kilogram-meter squared	I_total_kgm2	0.001	kg*m^2	
Natural Frequency in radians/sec	rb_nat_freq_rad_sec	10.998	rad/s	Mt converted to kg
Natural Frequency in Hz	rb_nat_freq_hz	1.750	Hz	
Period of Oscillation	rb_period	0.571	s	
Calculated time of one revolution of escapement wheel	rb_time_calc	7.998	s	

Figure 10: Time Calculation for Rigid-Body in Excel

3 Discussion

3.1 Experimental Results

After the pendulum clock was fabricated, the actual time required for one full revolution of the escapement wheel was found experimentally and the value is 9.810 seconds. WIth the other experimental values found, the percent errors could be calculated.

Calculated Mass of Acrylic	M_Calc	79.750	g
Actual Mass of Acrylic	M_Act	66.000	g
Percent Error of Mass Calculation	M_Error	-20.83%	

Figure 11: Mass Percent Error in Excel

Length to Center of Mass of Pendulum with nuts and bolts **In Meters**	Lcom_meter	0.079	**meters**	
Estimated Length to Center of Mass of Pendulum with nuts and bolts	Lcom_est	8.000	cm	recorded with brass bearing
Percent Error in Pendulum Nuts and Bolts	Lcom_error	2%		

Figure 12: Center of Mass Percent Error in Excel

The mass of the acrylic was found using a scale and the experimental value is 66 grams, giving a percent error of -20.83% (Figure 11). The center of mass was found experimentally by balancing the pendulum on a finger and recorded with a ruler. The actual value is 8

Calculated time of one revolution of escapement wheel	time_calc	7.877	s
Measured time of One Revolution of Escapement Wheel	time_meas	9.810	s
Percent Error in Clock Timing	time_error	19.70%	

Figure 13: Time Percent Error for Point-Mass in Excel

Calculated time of one revolution of escapement wheel	rb_time_calc	7.998	s	
Percent difference of point mass and rigid body assumption	per_diff	1.51%		
Percent Error in Clock Timing	rb_time_error	18%		

Figure 14: Time Percent Error for Rigid-Body in Excel

centimeters, giving a percent error of 2%. (Figure 12). The percent error between the time calculations for the point-mass and rigid-body method is 1.51% (Figure 14).

The percent error of the calculated and actual time for the point-mass method is 19.70% (Figure 13), while the percent error for the rigid-body method is 18% (Figure 14).

3.2 Mass Percent Error

The percent error is -20.83%. Possible sources of error for the actual mass to be much less than the calculated mass is due to defects in the acrylic. Impurities in the acrylic when it was manufactured could have caused the density of the acrylic sheet to be much less.

3.3 Center of Mass Percent Error

The percent error is 2%. Possible sources of error for the actual center of mass is the copper bushing. As I commented on in Figure 12, the center of mass was found after the insertion of the copper bushing.

3.4 Time Percent Error between Point-Mass and Rigid-Body

The percent error is 1.51%. For two different theoretical methods, these time values are really close to one another. When compared to the actual time value which is a larger, it makes more sense for that the rigid-body calculation is closer compared to point-mass. This shows that the rigid-body method is more accurate than point-mass.

3.5 Time Percent Error between Point-Mass and Actual Value

The percent error is 19.70%. Possible sources of error are friction and air resistance in the pendulum clock. The rubbing of the escapement wheel on the pendulum, as well as, the components on their neighboring components cause the actual time to be much larger than the calculated.

3.6 Time Percent Error between Rigid-Body and Actual Value

The percent error is 18%. Similar to the previous section, friction and air resistance are the most likely sources of error.

4 Appendix

The appendix shows the complete excel sheet used for calculations. The first sheet shows only the values, while the second one also shows formulas.

Pendulum Timing Analysis

Name: Roger Nguyen
Section: A04

Variable Description	Variable Name	Values/Equations	Units	Comments
Acrylic Pendulum Specifications				
Area	A	105.672	cm^2	
Thickness	t	0.635	cm	
Volume	Vol	67.102	cm^3	
Density	p	1.188	g/cm^3	
Calculated Mass of Acrylic	M_Calc	79.750	g	
Actual Mass of Acrylic	M_Act	66.000	g	
Percent Error of Mass Calculation	M_Error	-20.83%		
Length to Center of Mass of Acrylic	La	5.339	cm	
Calculate Total Mass of Pendulum				
Mass of one bolt and two nut (Bottom Bolts)	Mb2	4.000	g	given
Number of bolts with 2 nuts	N2	8		
Total mass of pendulum with adapter and bolts and nuts	Mt	111.750	g	calculated
Calculate Center of Mass of Pendulum with Bolts				
Length to Center of Mass of Bolt 1	L_bolt1	14.379	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 2	L_bolt2	14.379	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 3	L_bolt3	14.369	cm	distance from pivot point to bolt
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Length to Center of Mass of Pendulum with nuts and bolts **In Meters**	Lcom_meter	0.079	**meters**	
Estimated Length to Center of Mass of Pendulum with nuts and bolts	Lcom_est	8.000	cm	recorded with brass bearing
Percent Error in Pendulum Nuts and Bolts	Lcom_error	2%		

Calculate Natural Frequency and Timing using Point Mass Assumption				
Gravitational Constant	g	9.810	m/s^2	
Natural Frequency in radians/sec	nat_freq_rad_sec	11.167	rad/s	
Natural Frequency in Hz	nat_freq_hz	1.777	Hz	
Period of Oscillation	period	0.563	s	
Number of teeth of escapement wheel	nteeth	14		
Calculated time of one revolution of escapement wheel	time_calc	7.877	s	
Measured time of One Revolution of Escapement Wheel	time_meas	9.810	s	
Percent Error in Clock Timing	time_error	19.70%		

Calculate Natural Frequency and Timing using Rigid Body Assumption				
moment of inertia of acrylic pendulum	I_a	597.313	g*cm^2	
moment of inertia of bolt 1	I_bolt1	877.107	g*cm^2	
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moment of inertia of bolt 6	I_bolt6	713.958	g*cm^2	
moment of inertia of bolt 7	I_bolt7	827.483	g*cm^2	
moment of inertia of bolt 8	I_bolt8	827.483	g*cm^2	
total moment of inertia	I_total	7130.359	g*cm^2	
total moment of inertia in kilogram-meter squared	I_total_kgm2	0.001	kg*m^2	
Natural Frequency in radians/sec	rb_nat_freq_rad_sec	10.998	rad/s	Mt converted to kg
Natural Frequency in Hz	rb_nat_freq_hz	1.750	Hz	
Period of Oscillation	rb_period	0.571	s	
Calculated time of one revolution of escapement wheel	rb_time_calc	7.998	s	
Percent difference of point mass and rigid body assumption	per_diff	1.51%		
Percent Error in Clock Timing	rb_time_error	18%		

Pendulum Timing Analysis

Name: Roger Nguyen
Section: A04

Variable Description	Variable Name	Values/Equations	Units
Acrylic Pendulum Specifications			
Area	A	105.672	cm^2
Thickness	t	0.635	cm
Volume	Vol	=A*t	cm^3
Density	p	1.18849844	g/cm^3
Calculated Mass of Acrylic	M_Calc	=p*Vol	g
Actual Mass of Acrylic	M_Act	66	g
Percent Error of Mass Calculation	M_Error	=(M_Act - M_Calc) / M_Act	
Length to Center of Mass of Acrylic	La	5.33908	cm
Calculate Total Mass of Pendulum			
Mass of one bolt and two nut (Bottom Bolts)	Mb2	4	g
Number of bolts with 2 nuts	N2	8	
Total mass of pendulum with adapter and bolts and nuts	Mt	=M_Calc + (Mb2_* N2_)	g
Calculate Center of Mass of Pendulum with Bolts			
Length to Center of Mass of Bolt 1	L_bolt1	14.37894	cm
Length to Center of Mass of Bolt 2	L_bolt2	14.37894	cm
Length to Center of Mass of Bolt 3	L_bolt3	14.36878	cm
Length to Center of Mass of Bolt 4	L_bolt4	14.36878	cm
Length to Center of Mass of Bolt 5	L_bolt5	14.36878	cm
Length to Center of Mass of Bolt 6	L_bolt6	14.36878	cm
Length to Center of Mass of Bolt 7	L_bolt7	13.55344	cm
Length to Center of Mass of Bolt 8	L_bolt8	13.55344	cm
Length to Center of Mass of Pendulum with nuts and bolts	Lcom	=(M_Calc*La+Mb2_*(L_bolt1+L_bolt2+I	cm
Length to Center of Mass of Pendulum with nuts and bolts **In Meters**	Lcom_meter	=Lcom/100	**meters**
Estimated Length to Center of Mass of Pendulum with nuts and bolts	Lcom_est	8	cm
Percent Error in Pendulum Nuts and Bolts	Lcom_error	=(Lcom_est-Lcom)/Lcom_est	

Calculate Natural Frequency and Timing using Point Mass Assumption			
Gravitational Constant	g	9.81	m/s^2
Natural Frequency in radians/sec	nat_freq_rad_sec	=SQRT(g/Lcom_meter)	rad/s
Natural Frequency in Hz	nat_freq_hz	=nat_freq_rad_sec/(2*PI())	Hz
Period of Oscillation	period	=1/nat_freq_hz	s
Number of teeth of escapement wheel	nteeth	14	
Calculated time of one revolution of escapement wheel	time_calc	=period*nteeth	s
Measured time of One Revolution of Escapement Wheel	time_meas	9.81	s
Percent Error in Clock Timing	time_error	=(time_meas-time_calc)/time_meas	

Calculate Natural Frequency and Timing using Rigid Body Assumption			
moment of inertia of acrylic pendulum	I_a	597.313	g*cm^2
moment of inertia of bolt 1	I_bolt1	=Mb2_ * (14.808)^2	g*cm^2
moment of inertia of bolt 2	I_bolt2	=Mb2_ * (14.808)^2	g*cm^2
moment of inertia of bolt 3	I_bolt3	=Mb2_ * (14.56)^2	g*cm^2
moment of inertia of bolt 4	I_bolt4	=Mb2_ * (14.56)^2	g*cm^2
moment of inertia of bolt 5	I_bolt5	=Mb2_ * (13.36)^2	g*cm^2
moment of inertia of bolt 6	I_bolt6	=Mb2_ * (13.36)^2	g*cm^2
moment of inertia of bolt 7	I_bolt7	=Mb2_ * (14.383)^2	g*cm^2
moment of inertia of bolt 8	I_bolt8	=Mb2_ * (14.383)^2	g*cm^2
total moment of inertia	I_total	=SUM(C48:C56)	g*cm^2
total moment of inertia in kilogram-meter squared	I_total_kgm2	=I_total*10^-7	kg*m^2
Natural Frequency in radians/sec	rb_nat_freq_rad_sec	=SQRT((Mt/1000)*g*Lcom_meter/I_tota	rad/s
Natural Frequency in Hz	rb_nat_freq_hz	=rb_nat_freq_rad_sec/(2*PI())	Hz
Period of Oscillation	rb_period	=1/rb_nat_freq_hz	s
Calculated time of one revolution of escapement wheel	rb_time_calc	=rb_period*nteeth	s
Percent difference of point mass and rigid body assumption	per_diff	=(rb_time_calc-time_calc) / rb_time_calc	
Percent Error in Clock Timing	rb_time_error	=(time_meas-rb_time_calc)/time_meas	

References

[1] Fundamentals of Physics 11th ed, Halliday, David, et al. (2018)