Force-Torque-Power-Energy Analysis

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Contents

1 Abstract

The objective of this analysis is to determine the amount of torque, power, and energy required to pivot the upper joint around the motor's axis. The power source that will be used is the geared motor because it has the most torque. The factors of safety for energy and power both exceeded 3, while the factor of safety for torque was 1.449. No mechanical advantage is required.

2 Introduction

The powered part I will perform the force-torque-power-energy analysis on is the motor that will pitch the upper joint of the robotic arm. In figure 1, the black circle indicated by the text "motor" is the location of the motor that I will perform this analysis on.

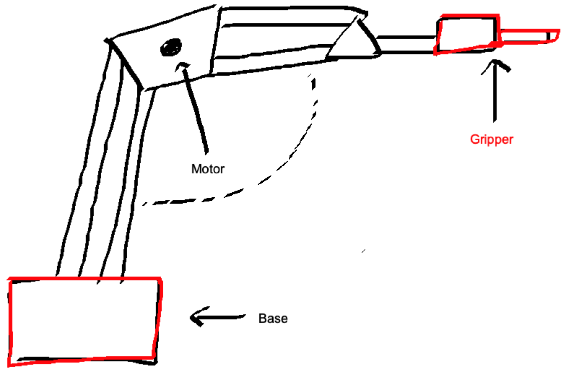

Figure 1: My Part

In figure 2, I included the more detailed general design of the robotic arm to better showcase the bigger picture of how the robotic arm will look like.

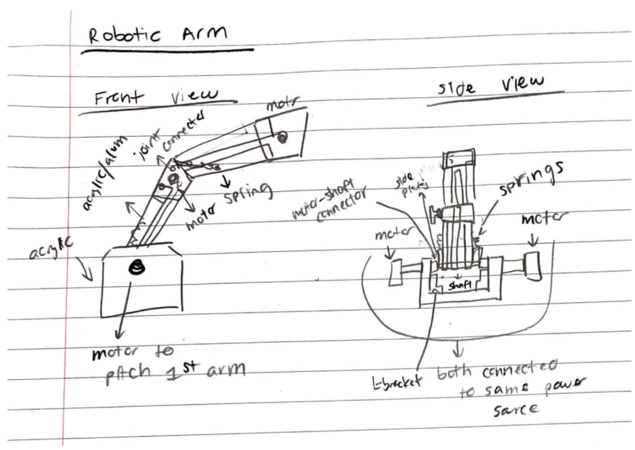

Figure 2: General Design

In figure 3, a simplified model of the arm has been drawn. The upper joint is expected to rotate about a pivot point from an initial position (red rectangle) and a final position (blue rectangle). The variable L denotes the length from the pivot point to the center of mass of the upper joint.

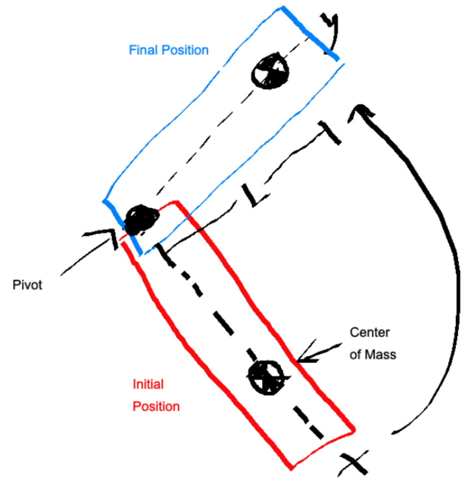

Figure 3: Simplified Model

3 Calculations

3.1 Design Parameters

Mass of Upper Joint	0.20 kg
Length from Pivot to Center of Mass	0.1 m
Angle of Rotation	90 degrees
Time to Move Arm	2 seconds

3.2 Assumptions

- Friction and air resistance is ignored
- The upper joint rotates at a constant angular acceleration
- The upper joint is simplified to a thin rod

3.3 Energy Analysis

The amount of energy required to rotate the arm is dependent on the angular velocity and the rotational inertia, using the equation $E_{rot} = \frac{1}{2}I\omega^2$ [1].

The rotational inertia around the pivot point of the arm can be calculated using the equation $I = I_{com} + Md^2$ [1]. Since the upper joint is assumed to be a rod, the rotational inertia about the center of mass is $I_{com} = \frac{1}{12}M(\frac{3L}{2})^2$ [1]. However, since the upper joint isn't rotating about its center of axis as shown in figure 3, we will have to use the parallel axis theorem. The rotational inertia equation can be written out as

$$I = \frac{1}{12} M \left(\frac{3L}{2}\right)^2 + ML^2$$
$$I = \frac{1}{12}(0.5kg)\left(\frac{3(0.1m)}{2}\right)^2 + (0.5kg)(0.1m)^2$$
$$I = 0.00594 \text{ kg-}m^2$$

For this calculation, we only want the average angular velocity and the upper joint will rotate at this angular velocity during the entire angular displacement. The equation for average angular velocity can be written out as

$$\omega_{avg} = \frac{\Delta\theta}{\Delta t}[1]$$
$$\omega_{avg} = \frac{\pi/2}{2s}$$
$$\omega_{avg} = 0.785 \text{ rad/s}$$

The energy required to rotate the upper joint can be calculated using

$$E_{rot} = \frac{1}{2} I\omega^2[1]$$
$$E_{rot} = \frac{1}{2}(0.00594 \text{ kg-}m^2)(0.785 \text{ rad/s})^2$$
$$E_{rot} = 0.00183J$$

3.4 Power Analysis

The energy needed to rotate the is 0.00183 Joules. Using the equation for power, the power required can be calculated as

$$P = \frac{E}{t}[1]$$
$$P = \frac{0.00183J}{2s}$$
$$P = 0.000915W$$

3.5 Torque Analysis

In figure 4, the worst case load scenario for the upper joint has been drawn as a free-body diagram. The reason for this is because the force of gravity will create the greatest torque opposing the rotation of the joint when it is horizontal. In the following calculations, this worst case scenario will be used to determine the amount of torque needed.

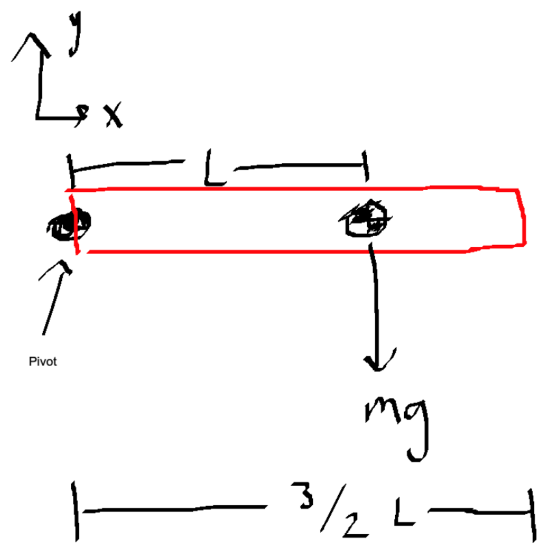

Figure 4: Worse Case Scenario

The maximum amount of torque due to the force of gravity can be found using the equation

$$\tau_{mg} = r \times F_{mg}[1]$$
$$\tau_{mg} = L * mgsin(\theta)$$
$$\tau_{mg} = (0.1m) * (0.2kg)(9.8m/s^2)sin(90)$$
$$\tau_{mg} = 0.196Nm$$

Thus, the minimum amount of torque required is 0.196 Newton-meters.

4 Selecting Power Source

In the Team Force-Torque-Energy-Power assignment, the energies, powers, and torques of each of the possible power sources were found experimentally. The summary of this lab can be seen in figure 5.

Summary of Energy Sources:

	Max Force/Torque:	Max Energy:	Max Power:
Geared Motor	0.284 Nm	29.33 J	0.49061 W
Non-Geared Motor	0.00866 Nm	40.81 J	0.6802 W
Small Rubber Band	5.61 N	0.356 J	2.37 W
Med Rubber Band	5.34 N	0.281 J	1.35 W
Large Rubber Band	5.17 N	0.151 J	0.888 W
Small Spring	10.51 N	0.704 J	3.71 W
Medium Spring	17.31 N	0.918 J	6.56 W
Large Spring	29.33 N	1.554 J	10.4 W
Weight (Pot. Energy)	6.145 N	1.561 J	6.88 W

	Component:	Value:
Max Force:	Large Spring	29.33 N
Max Torque:	Geared Motor	0.284 Nm
Max Power:	Weight	6.88 W
Max Energy:	Geared Motor	40.81 J

Figure 5: Summary of Max Energies, Powers, and Torques

Due to the torque requirement, I decided to use the geared motor to pivot the upper joint. The values of the geared motored can be seen in figure 6. The energy and power requirements are both satisfied by the geared motor; however, the amount of torque isn't enough and a mechanical advantage will need to be implemented in order to generate enough torque to rotate the upper joint.

		Geared Motor
Stall Angle (deg):	θ	50 deg
Moment Arm Length (m):	L	13 cm = 0.13 m
Mass (kg):	m	291 g = .291 kg
Stall Torque (Nm):	$\tau_{stall} = mgLsin\theta$	0.284 Nm
No Load Speed (rad/s):	ω_{noLoad}	1.1 rev/s = 6.91 rad/s
[redacted]		
Max Power (watts):	$P = \frac{1}{2}\tau_{stall} * \frac{1}{2}\omega_{noLoad}$	0.49061 W
Max Energy in 60s (J):	$E = P \cdot t$	29.33 J

Figure 6: Geared Motor Values

4.1 Factor of Safety

The equation for factor of safety can be generalized to $FOS = \frac{Have}{Required}$.

The factor of safety for energy is $FOS_E = \frac{0.98122J}{0.00183J} = 536$.

The factor of safety for power is $FOS_P = \frac{0.49061W}{0.000915W} = 536$.

The factor of safety for torque is $FOS_\tau = \frac{0.284Nm}{0.196Nm} = 1.449$.

5 Conclusion

After performing a force-torque-power-energy analysis on the motor for the upper joint, I found that 0.00183 Joules of Energy, 0.000915 Watts of Power, and 0.196 Newton-meters of torque was needed. With these values, I decided to choose the geared motor. The reasoning for this is that the torque needed without a mechanical advantage would be the geared motor.

References

[1] Fundamentals of Physics 11th ed, Halliday, David, et al. (2018)